If we want gradients for only a few g's, the second approach is clearly most efficient. Arora and Haug make the assumption that NG is always small compared to NDV*NLC. In a real design situation where we are setting up an approximate design problem, either for hand calculations or for optimization, this may not be true since NG will be the number of constraints *retained for design consideration*. In any case, the message is that the order of the matrix operation should be determined at the time it is done, based upon a comparison of NG and NDV*NLC and the computational cost of each ordering. In terms of *total* computational effort to analyze the structure and compute gradients, the time difference should not be as significant as we are led to believe.

Based upon the discussion, the following conclusions seem to be in order;

1) A gradient is a gradient, no matter how it is calculated. In suggesting that their method is more general, Arora and Haug are only saying that they retain more terms. Certainly the referenced investigators are qualified to do the same if they need the additional terms.

2) The conclusion that their method is up to ten times faster is valid so long as we understand that they are talking about one *part* of the total computational effort.

3) It is important to remember that the order of a matrix operation is criticial to the computational efficiency of performing that operation.

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Reply by Authors to G.N. Vanderplaats

Jasbir S. Arora* and Edward J. Haug†
The University of Iowa, Iowa City, Iowa

IN responding to Vanderplaats' Comment, we would first like to agree with his operations count and the observation that the order of matrix computation can lead to a computational advantage of up to a factor of ten for the state space method over the design space method for "one part of the computational effort." It is worth noting that for many constraints, such as displacement and stress constraints for constant strain elements, this "one part of the computational effort" is the only computational effort needed for gradient calculations.

There are some points in Vanderplaats' Comments with which we disagree. First, the main point of our paper was that the state space method, which has been a principal tool of optimal control theory for two decades,[1,2] unifies design sensitivity analysis in the field of structural optimization. Our paper shows that the excellent design derivative analysis method of Venkayya and co-workers[3-9] can be viewed as a special case of the adjoint variable, state-space method of design sensitivity analysis. Thus, the unified state space method developed by Bryson[2] for optimal control and adopted by the writers[10] provides a general approach for design sensitivity analysis in a wide variety of structural and mechanical system design problems. Vanderplaats' interpretation of the state space method of design sensitivity analysis as "a science of chain rule differentiation" is unfortunate and worthy of clarification.

In addition to being directly applicable for static response problems, which was the focus of our paper, the method leads directly to an algorithm for design sensitivity analysis and optimization of structures and vibration isolators under dynamic loads.[11-13] The method is also ideally suited for incorporation with the substructuring method of structural analysis.[10,14] More recently, the method has been applied for design sensitivity analysis of large-scale, nonlinear mechanisms and machines.[15] Finally, the state space method is directly applicable to structures whose displacement is the solution of partial differential equations.[10,13,16] For such distributed parameter problems, the state space approach outlined in our paper is used with integral scalar product for design sensitivity analysis of systems described by differential equations, which generalize the matrix method of our paper. Thus, the state space method we present carries over to general structural and mechanical systems described by differential equations, whereas the direct differentiations used in the design space method are meaningless or impractical. Thus, much as in the case of optimal control theory,[2] there is indeed a systematic theory of design sensitivity analysis of broad classes of structural and mechanical systems, which is hardly as simplistic as a "science of chain-rule differentiation."

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Index categories: Structural Design; Analytical and Numerical Methods; Structural Statics.

*Associate Professor.

†Professor, Materials Division, College of Engineering.

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Comment on "Criterion for Vortex Periodicity in Cylinder Wakes"

N. R. Keshavan*
Hindustan Aeronautics Limited, Bangalore, India

IN reviewing many papers, Ericsson and Reding[1] have concluded in their Note that a necessary condition for the establishment of vortex periodicity in cylinder wakes is the existence of a well-defined, two-dimensional separated flow region. They have suggested also that the periodic vortex shedding with the associated problems of self-excited oscillations could be eliminated by introducing three-dimensional flow disturbances that prevent the formation of a well-defined, two-dimensional flow separation geometry.

Ericsson and Reding[1] have omitted an important paper by Naumann et al.,[2] where the same conclusions are drawn based on experimental results. Naumann et al. have experimentally studied the effect of artificially forcing separation of the flow on cylinders by means of separation wires and observed that the periodic vortex shedding could be avoided by means of zigzag separation wires along the cylinder span. Based on experimental results, they have also given a criterion for the minimum amount of three-dimensional disturbance necessary to avoid periodic shedding of vortices.

Through acoustic measurements, Keshavan[3] has also shown that by forcing nonlinear spanwise separation of the flow on circular cylinders, one can avoid periodic vortex shedding and the associated noise. The nonlinear separation of the flow along the span of the cylinder was achieved by blowing through three tangential slots. The three tangential spanwise slots used were of different lengths along the span and located at different azimuthal angles. This arrangement of slots produced an irregular separation line resulting in the strong attenuation of periodic vortex shedding noise.

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Index categories: Jets, Wakes and Viscid-Inviscid Flow Interactions; Nonsteady Aerodynamics.

*Design Engineer.

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Reply by Authors to N. R. Keshavan

L. E. Ericsson* and J. P. Reding†
Lockheed Missiles & Space Company, Inc., Sunnyvale, Calif.

IN his Comment to our Technical Note,[1] Keshavan quite correctly points out that the work by Naumann et al.,[2] which we for some reason did not uncover in our literature search,[3] has important implications in regard to our conclusions. The detailed experimental research by Naumann et al.[2] shows conclusively that a well defined two-dimensional flow separation is needed for the establishment of a Karman vortex street. The main point of our Note is that the absence of vortex periodicity in the critical Reynolds number region is due to a lack of a two-dimensionality, caused by the spanwise variation of the boundary layer transition. How this leads to an absence of self-excited oscillations is discussed in more detail in Ref. 4.

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*Consulting Engineer. Associate Fellow AIAA.
†Research Specialist. Member AIAA.